

Practice Test - Chapter 8

Find the geometric mean between each pair of numbers.

1. 7 and 11

SOLUTION:

By the definition, the geometric mean x of any two numbers a and b is given by $x = \sqrt{ab}$.

Therefore, the geometric mean of 7 and 11 is

$$\sqrt{(7)(11)} = \sqrt{77} \approx 8.8.$$

ANSWER:

$$\sqrt{77} \approx 8.8$$

2. 12 and 9

SOLUTION:

By the definition, the geometric mean x of any two numbers a and b is given by $x = \sqrt{ab}$.

Therefore, the geometric mean of 12 and 9 is

$$\sqrt{(12)(9)} = 6\sqrt{3} \approx 10.4.$$

ANSWER:

$$6\sqrt{3} \approx 10.4$$

3. 14 and 21

SOLUTION:

By the definition, the geometric mean x of any two numbers a and b is given by $x = \sqrt{ab}$.

Therefore, the geometric mean of 14 and 21 is

$$\sqrt{(14)(21)} = 7\sqrt{6} \approx 17.1.$$

ANSWER:

$$7\sqrt{6} \approx 17.1$$

4. $4\sqrt{3}$ and $10\sqrt{3}$

SOLUTION:

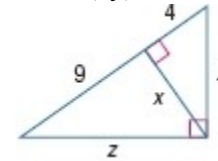
By the definition, the geometric mean x of any two numbers a and b is given by $x = \sqrt{ab}$.

Therefore, the geometric mean of $4\sqrt{3}$ and $10\sqrt{3}$ is

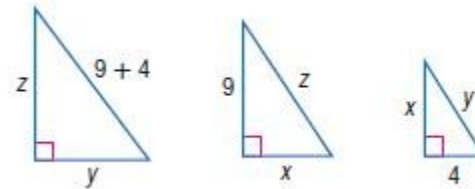
$$\sqrt{(4\sqrt{3})(10\sqrt{3})} = 2\sqrt{30} \approx 11.$$

$$2\sqrt{30} \approx 11.0$$

Find x , y , and z .



SOLUTION:



$$\frac{\text{hypotenuse}}{\text{longer leg}} = \frac{\text{hypotenuse}}{\text{longer leg}} \quad \frac{\text{hypotenuse}}{\text{shorter leg}} = \frac{\text{hypotenuse}}{\text{shorter leg}}$$

Set up a proportion and solve for z and y :

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$$\frac{13}{z} = \frac{z}{9} \quad \text{and} \quad \frac{13}{y} = \frac{y}{4}$$

$$\begin{aligned} z^2 &= 117 & y^2 &= 52 \\ z &= \sqrt{117} & \text{and } y &= \sqrt{52} \\ &= 3\sqrt{13} & &= 2\sqrt{13} \end{aligned}$$

By the Geometric Mean (Altitude) Theorem the altitude drawn to the hypotenuse of a right triangle separates the hypotenuse into two segments. The length of this altitude is the geometric mean between the lengths of these two segments.

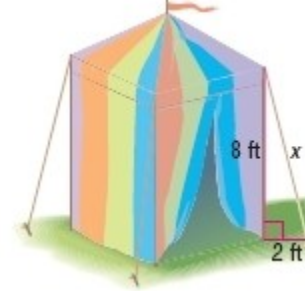
Solve for x

$$\begin{aligned} \frac{9}{x} &= \frac{x}{4} \\ x^2 &= 36 \\ x &= 6 \end{aligned}$$

ANSWER:

$$x = 6, y = 2\sqrt{13}, z = 3\sqrt{13}$$

6. **FAIRS** Blake is setting up his tent at a renaissance fair. If the tent is 8 feet tall, and the tether can be staked no more than two feet from the tent, how long should the tether be?



SOLUTION:

Use the Pythagorean Theorem to find x .

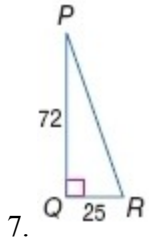
$$\begin{aligned} 2^2 + 8^2 &= x^2 \\ 4 + 64 &= x^2 \\ 68 &= x^2 \\ \sqrt{68} &= x \\ x &= 2\sqrt{17} \approx 8.2 \text{ ft} \end{aligned}$$

ANSWER:

8.2 ft

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Use a calculator to find the measure of $\angle R$ to the nearest tenth.



SOLUTION:

The measures given are those of the legs opposite and adjacent to $\angle R$, so write an equation using the tangent ratio.

$$\tan R = \frac{PQ}{QR} = \frac{72}{25} = 2.88.$$

$$\text{If } \tan R = 2.88, \text{ then } R = \tan^{-1}(2.88).$$

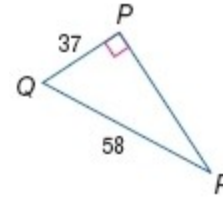
Use a calculator.

A calculator screen showing the calculation of the inverse tangent of 2.88. The display shows $\tan^{-1}(2.88)$ and the result 70.85186254.

$$m\angle R \approx 70.9^\circ$$

ANSWER:

70.9



8.

SOLUTION:

The measures given are those of the leg opposite to $\angle R$ and the hypotenuse, so write an equation using the sine ratio.

$$\sin R = \frac{PQ}{QR} = \frac{37}{58}$$

$$\text{If } \sin R = \frac{37}{58}, \text{ then } \sqrt{T} = \sin^{-1}\left(\frac{37}{58}\right).$$

Use a calculator.

A calculator screen showing the calculation of the inverse sine of 37/58. The display shows $\tan^{-1}(2.88)$ and the result 70.85186254.

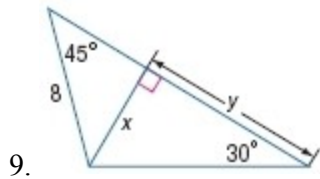
$$m\angle R \approx 39.6^\circ$$

ANSWER:

39.6

Find x and y.

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SOLUTION:

$$\sin 45^\circ = \frac{x}{8}$$

$$x = 8 \sin 45^\circ$$

$$x = 8 \left(\frac{1}{\sqrt{2}} \right)$$

$$x = \frac{8}{\sqrt{2}}$$

$$x = \frac{8 \cdot \sqrt{2}}{\sqrt{2} \cdot \sqrt{2}}$$

$$x = \frac{8\sqrt{2}}{2}$$

$$x = 4\sqrt{2}$$

$$\tan 30^\circ = \frac{x}{y}$$

$$\frac{1}{\sqrt{3}} = \frac{4\sqrt{2}}{y}$$

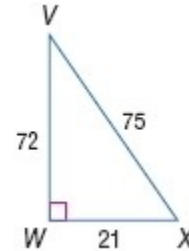
$$y = 4\sqrt{2} \cdot \sqrt{3}$$

$$y = 4\sqrt{6}$$

ANSWER:

$$x = 4\sqrt{2}, y = 4\sqrt{6}$$

Express each ratio as a fraction and as a decimal to the nearest hundredth.



10. $\cos X$

SOLUTION:

The cosine of an angle is defined as the ratio of the adjacent side to the hypotenuse. So,

$$\cos X = \frac{WX}{VX} = \frac{21}{75} = 0.28.$$

ANSWER:

$$\frac{21}{75} = 0.28$$

11. $\tan X$

SOLUTION:

The tangent of an angle is defined as the ratio of the opposite side to the adjacent side. So,

$$\tan X = \frac{VW}{WX} = \frac{72}{21} \approx 3.43.$$

ANSWER:

$$\frac{72}{21} \approx 3.43$$

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12. $\tan V$

SOLUTION:

The tangent of an angle is defined as the ratio of the opposite side to the adjacent side. So,

$$\tan V = \frac{WX}{VW} = \frac{21}{72} \approx 0.29.$$

ANSWER:

$$\frac{21}{72} \approx 0.29$$

13. $\sin V$

SOLUTION:

The sine of an angle is defined as the ratio of the opposite side to the hypotenuse. So,

$$\sin V = \frac{WX}{VX} = \frac{21}{75} = 0.28.$$

ANSWER:

$$\frac{21}{75} = 0.28$$

Find the magnitude and direction of each vector.

14. \overline{JK} : $J(-6, -4)$ and $K(-10, -4)$

SOLUTION:

Use the distance formula to find the vector's magnitude.

$$\begin{aligned} \text{magnitude of } \overline{JK} &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(-10 + 6)^2 + (-4 + 4)^2} \\ &= \sqrt{(-4)^2} \\ &= 4 \end{aligned}$$

Use trigonometry to find the vector's direction.

$$\begin{aligned} \tan J &= \left| \frac{y_2 - y_1}{x_2 - x_1} \right| \\ &= \left| \frac{-4 + 4}{-10 + 6} \right| \\ &= \left| \frac{0}{-4} \right| \\ &= 0 \\ J &= \tan^{-1}(0) \\ &= 0^\circ \end{aligned}$$

When repositioned so that its initial point is at the origin, \overline{JK} lies on the negative x -axis and forms an angle of 0° . So, the direction of \overline{JK} is the angle it makes with the positive x -axis which is $0^\circ + 180^\circ$ or 180° .

ANSWER:

4 units; 180°

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15. \overline{RS} : $R(1, 0)$ and $S(-2, 3)$

SOLUTION:

Use the distance formula to find the vector's magnitude.

$$\begin{aligned}\text{magnitude of } \overline{RS} &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(-2 - 1)^2 + (3 - 0)^2} \\ &= \sqrt{(-3)^2 + (3)^2} \\ &= \sqrt{9 + 9} \\ &= \sqrt{18} \\ &= 3\sqrt{2}\end{aligned}$$

Use trigonometry to find the vector's direction.

$$\begin{aligned}\tan R &= \left| \frac{y_2 - y_1}{x_2 - x_1} \right| \\ &= \left| \frac{-2 - 1}{3 - 0} \right| \\ &= \left| \frac{-3}{3} \right| \\ &= 1 \\ R &= \tan^{-1}(1) \\ &= 45^\circ\end{aligned}$$

When repositioned so that its initial point is at the origin, \overline{RS} lies in the same quadrant and forms an angle with the positive y -axis 45° counter clockwise.

So, the direction of \overline{RS} is the angle it makes with the positive x -axis counter clockwise, which is $90^\circ + 45^\circ$ or 135° .

ANSWER:

$$3\sqrt{2}, 135^\circ$$

16. **SPACE** Anna is watching a space shuttle launch 6 miles from Cape Canaveral in Florida. When the angle of elevation from her viewing point to the shuttle is 80° , how high is the shuttle, if it is going straight up?

SOLUTION:

Let x be the unknown.

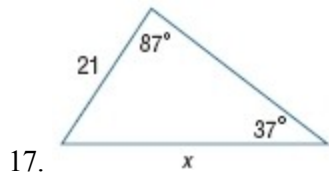
$$\begin{aligned}\tan 80^\circ &= \frac{x}{6} \\ x &= 6 \tan 80^\circ \\ x &\approx 34 \text{ mi}\end{aligned}$$

ANSWER:

34 mi

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Find x . Round angle measures to the nearest degree and side measures to the nearest tenth.



SOLUTION:

By the Law of Sines, $\frac{\sin 37^\circ}{21} = \frac{\sin 87^\circ}{x}$.

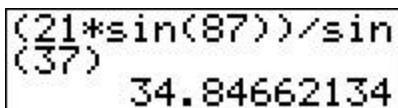
Take the cross products.

$$x \cdot \sin 37^\circ = 21 \cdot \sin 87^\circ$$

Divide each side by $\sin 37^\circ$.

$$x = \frac{21 \cdot \sin 87^\circ}{\sin 37^\circ}$$

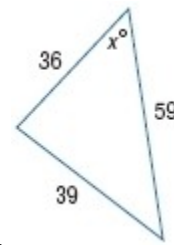
Use a calculator, in degree mode.



$$x \approx 34.8$$

ANSWER:

34.8



SOLUTION:

We are given the measures of two sides and their included angle, so use the Law of Cosines.

$$39^2 = 36^2 + 59^2 - 2(36)(59)\cos x$$

$$1521 = 1296 + 3481 - 4248\cos x$$

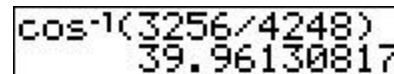
$$1521 = 4777 - 4248\cos x$$

$$4248\cos x = 4777 - 1521$$

$$4248\cos x = 3256$$

$$\cos x = \frac{3256}{4248}$$

$$x = \cos^{-1}\left(\frac{3256}{4248}\right)$$



$$x \approx 40.0$$

ANSWER:

40

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19. **MULTIPLE CHOICE** Which of the following is the length of the leg of a $45^\circ - 45^\circ - 90^\circ$ triangle with a hypotenuse of 20?

A 10
B $10\sqrt{2}$
C 20
D $20\sqrt{2}$

SOLUTION:

Let x be the unknown.
Use the Pythagorean Theorem.

$$x^2 + x^2 = 20^2$$

$$2x^2 = 400$$

$$x^2 = 200$$

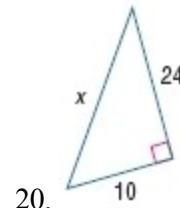
$$x = 10\sqrt{2}$$

So, the correct choice is B.

ANSWER:

B

Find x .



SOLUTION:

Use the Pythagorean Theorem to solve for x .

$$10^2 + 24^2 = x^2$$

$$100 + 576 = x^2$$

$$676 = x^2$$

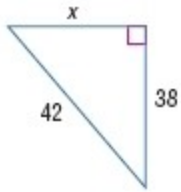
$$\sqrt{676} = x$$

$$x = 26$$

ANSWER:

26

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21.

SOLUTION:

Use the Pythagorean Theorem to solve for x .

$$x^2 + 38^2 = 42^2$$

$$x^2 + 1444 = 1764$$

$$x^2 = 1764 - 1444$$

$$x^2 = 320$$

$$x = \sqrt{320} \approx 17.9$$

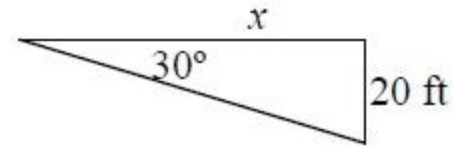
ANSWER:

$$\sqrt{320} \approx 17.9$$

22. **WHALE WATCHING** Isaac is looking through binoculars on a whale watching trip when he notices a sea otter in the distance. If he is 20 feet above sea level in the boat, and the angle of depression is 30° , how far away from the boat is the otter to the nearest foot?

SOLUTION:

Let x be the unknown.



$$\tan 30^\circ = \frac{20}{x}$$

$$x = \frac{20}{\tan 30^\circ}$$

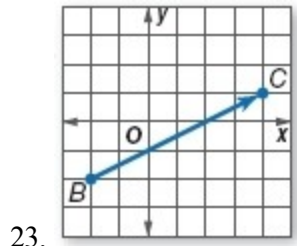
$$x \approx 35 \text{ ft}$$

ANSWER:

35 ft

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Write the component form of each vector.



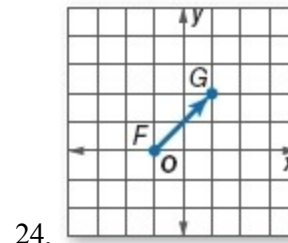
SOLUTION:

Use component notation and find the change in x -values and the change in y -values. Here, the coordinates of B and C are $(-2, -2)$ and $(4, 1)$ respectively.

$$\begin{aligned}\overrightarrow{AB} &= \langle x_2 - x_1, y_2 - y_1 \rangle \\ &= \langle 4 - (-2), 1 - (-2) \rangle \\ &= \langle 6, 3 \rangle\end{aligned}$$

ANSWER:

$$\langle 6, 3 \rangle$$



SOLUTION:

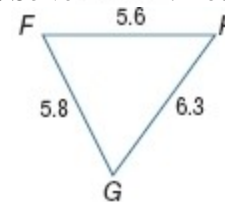
Use component notation and find the change in x -values and the change in y -values. Here, the coordinates of F and G are $(-1, 0)$ and $(1, 2)$ respectively.

$$\begin{aligned}\overrightarrow{AB} &= \langle x_2 - x_1, y_2 - y_1 \rangle \\ &= \langle 1 - (-1), 2 - 0 \rangle \\ &= \langle 2, 2 \rangle\end{aligned}$$

ANSWER:

$$\langle 2, 2 \rangle$$

25. Solve $\triangle FGH$. Round to the nearest degree.



SOLUTION:

Since we are given three sides and no angle measures, we can solve this triangle using the Law of Cosines.

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$$\begin{aligned}6.3^2 &= 5.6^2 + 5.8^2 - 2(5.6)(5.8)\cos F \\6.3^2 - (5.6^2 + 5.8^2) &= -64.96\cos F \\ \frac{6.3^2 - (5.6^2 + 5.8^2)}{-64.96} &= \cos F \\\cos^{-1}\left[\frac{6.3^2 - (5.6^2 + 5.8^2)}{-64.96}\right] &= F \\67.1 &\approx F\end{aligned}$$

Similarly, we can solve for the measure of angle G, using the Law of Cosines.

$$\begin{aligned}5.6^2 &= 6.3^2 + 5.8^2 - 2(6.3)(5.8)\cos G \\5.6^2 - (6.3^2 + 5.8^2) &= -73.08\cos G \\ \frac{5.6^2 - (6.3^2 + 5.8^2)}{-73.08} &= \cos G \\\cos^{-1}\left[\frac{5.6^2 - (6.3^2 + 5.8^2)}{-73.08}\right] &= G \\54.9 &\approx G\end{aligned}$$

We know that the sum of the measures all interior angles of a triangle is 180.

$$m\angle F + m\angle G + m\angle H = 180$$

$$67 + 55 + m\angle H = 180$$

$$m\angle H = 58$$

ANSWER:

$$m\angle F = 67^\circ, m\angle G = 55^\circ, m\angle H = 58^\circ$$